

Question Paper with Final (Revised) Answer Key for the Post of	Lecturer, Applied Science & Humanities (Polytechnic) Mathematics	held on 27-03-2021
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Itemcode : **HT1001**

Q1: Which one of the following is true?

- | | |
|----------|---|
| A | Every first countable topological space is separable. |
| B | The product of two Hausdroff spaces may not be a Hausdroff space. |
| C | Every compact Hausdroff space is connected. |
| D | Every finite T_2 space is discrete. |

Correct Ans: **D**

Itemcode : **HT1002**

Q2: Which one of the following is true?

- | | |
|----------|--|
| A | A Banach space need not be closed. |
| B | A closed subspace of Banach space is complete. |
| C | In a Banach space, every absolutely convergent series need not be convergent. |
| D | The normed linear space $(\mathbb{R}^n, \ \cdot\)$ is not complete where $\ \cdot\ $ is an arbitrary norm on $\mathbb{R}^n$. |

Correct Ans: **B**

Itemcode : **HT1003**

Q3: The order of convergence of Newton Raphson method is

- | | |
|----------|---|
| A | 1 |
| B | 2 |
| C | 3 |
| D | 4 |

Correct Ans: **B**

Itemcode : **HT1004**

Q4: Let $p(x)$ be a quadratic polynomial that takes the value $y(1) = 1$, $y(2) = 2$, $y(3) = 10$, then the value of $y(4)$ by Newton's forward difference formula is

- | | |
|----------|----|
| A | 17 |
| B | 18 |
| C | 20 |
| D | 25 |

Correct Ans: **D**

Itemcode : **HT1005**

Q5: By using Euler's method, the value of y when $x = 0.1$, given that $y(0) = 1$ and $y' = xy + y^2$

- | | |
|----------|------|
| A | 1.01 |
| B | 1.1 |
| C | 1.2 |
| D | 1.3 |

Correct Ans: **B**

Itemcode : **HT1006**

Q6: For the points (1,8) and (3,9), the linear polynomial P(x) that pass through these two points is

A $\frac{15}{2} + \frac{x}{2}$

B $\frac{5}{2} + \frac{x}{2}$

C $\frac{15}{2} - \frac{x}{2}$

D $15 - \frac{x}{3}$

Correct Ans: **A**

Itemcode : **HT1007**

Q7: If X is a continuous random variable whose probability distribution function is given by

$$f(x) = k(x - x^2), 0 \leq x \leq 1, \text{ then } P\left(X > \frac{1}{2}\right) \text{ is}$$

A 1/6

B 1/24

C $\frac{1}{2}$

D 1

Correct Ans: **C**

Itemcode : **HT1008**

Q8: The residue of the function

$$f(z) = \frac{1 + \cos(z)}{e^z - 1} \text{ at } z = 0 \text{ is}$$

A 1

B 2

C -1

D 0

Correct Ans: **B**

Itemcode : **HT1009**

Q9: The radius of convergence of the series $\sum \frac{e^n}{n} z^n$ is

A e

B e^{-1}

C ∞

D 0

Correct Ans: **B**

Itemcode : HT1010

Q10: The function $f(z) = \frac{\cos(\frac{\pi z}{2})}{\sin(\pi z)}$ has poles at

- A all odd integers
- B all even integers
- C all integers
- D all integers of the form $3k + 1$, k an integer

Correct Ans: B

Itemcode : HT1011

Q11: If $|z - 1| = |z - 2|$, then

- A $\operatorname{Re}(z) = 0$
- B $\operatorname{Re}(z) = 1$
- C $\operatorname{Re}(z) = 1/2$
- D $\operatorname{Re}(z) = 3/2$

Correct Ans: D

Itemcode : HT1012

Q12: Let $f: \mathbb{D} \rightarrow \mathbb{D}$ be a holomorphic function with $f(0) = 0$ where $\mathbb{D}$ is a unit disc $\{z: |z| < 1\}$. Then which one of the following is not correct?

- A $|f(1/4)| \leq 3/4$
- B $|f(3/4)| \leq 1/2$
- C $|f(1/4)| \leq 1/2$
- D $|f(1/2)| \leq 1/2$

Correct Ans: B

Itemcode : HT1013

Q13: The solution of initial value problem $y'' - 10y' + 21 = e^{3x}$, $y(0) = y'(0) = 0$ is

- A $y = \frac{-1}{8}e^{3x} + \frac{1}{8}e^{7x} - \frac{x}{7}e^{3x}$
- B $y = \frac{-1}{16}e^{3x} + \frac{1}{16}e^{7x} - \frac{x}{4}e^{3x}$
- C $y = \frac{-1}{8}e^{3x} + \frac{1}{8}e^{7x} - \frac{x^2}{7}e^{3x}$
- D $y = \frac{-1}{16}e^{3x} + \frac{1}{16}e^{7x} - \frac{x^2}{7}e^{3x}$

Correct Ans: B

Itemcode : HT1014

Q14:

The Laplace transform of the function $f(t) = 8e^{3t} \sin(\sqrt{3}t) - e^{5t}$ is

A $\frac{8(s-\sqrt{3})}{(s-\sqrt{3})^2+\sqrt{3}} - \frac{1}{s+5}$

B $\frac{8(s-3)}{(s-3)^2+\sqrt{3}} - \frac{1}{s-5}$

C $\frac{8(\sqrt{3})}{(s-3)^2+3} - \frac{1}{s-5}$

D $\frac{8(s-\sqrt{3})}{(s-\sqrt{3})^2+3} - \frac{1}{s+5}$

Correct Ans: **C**

Itemcode : **HT1015**

Q15: Find the inverse Laplace transform of the function $F(s) = \frac{s}{(s^2+1)^2}$.

A $\cos(t)$

B $2t\sin(t)$

C $(t\sin(t))/2$

D $(t\cos(t))/2$

Correct Ans: **C**

Itemcode : **HT1016**

Q16: The solution of initial value problem $y' = 2 - t$, $y(0) = 1$ is

A $y = 1 - 2t + \frac{t^2}{2}$

B $y = 1 + \frac{t}{2} - t^2$

C $y = 1 - \frac{t}{2} + t^2$

D $y = 1 + 2t - \frac{t^2}{2}$

Correct Ans: **D**

Itemcode : **HT1017**

Q17: The total work done by the force $\vec{F} = z\hat{i} - x\hat{j} + y\hat{k}$ moving a particle from (0,0,0) to (1,2,4) along a line segment is

A 10

B 5

C 15

D 8

Correct Ans: **B**

Itemcode : **HT1018**

Q18: The directional derivative of $f(x, y) = xe^y$ at $(1, 0)$ in the direction of the vector $\hat{i} + \hat{j}$ is

A $\frac{1}{\sqrt{2}}$

B 2

C $\sqrt{2}$

D $2\sqrt{2}$

Correct Ans: **C**

Itemcode : **HT1019**

Q19: The area lying between the parabola $y = 4x^2$ and the line $y = x$ is

A 1/16

B 1/6

C 1/32

D 1/96

Correct Ans: **D**

Itemcode : **HT1020**

Q20: The value of $\iint_S \vec{F} \cdot \hat{n} \, dS$ where $\vec{F} = x\hat{i} - y^2\hat{j} + z\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ is

A 2

B 3

C 4

D 6

Correct Ans: **B**

Itemcode : **HT1021**

Q21: The value of $\lim_{x \rightarrow \infty} \frac{\left(1 + \frac{1}{x}\right)^x - 1}{e}$ is

A 1/e

B -1/e

C (1/e) - 1

D 1 - (1/e)

Correct Ans: **D**

Itemcode : **HT1022**

Q22: The total number of one to one functions from $\{0, 1\}$ to $\{1, 2, 3, 4, 5\}$ is

A 2^5

B 20

C 5!

D	10
Correct Ans: B	

Itemcode : HT1023	
Q23: The sum of the series is $1 + \frac{1}{2!} + \frac{1}{2} + \frac{1}{3!} - \frac{1}{3} + \frac{1}{4!} + \frac{1}{4} + \dots$ equals	
A	$e - \log(2)$
B	$-e + \log(2)$
C	$e - \log(2) + 2$
D	$-e + \log(2) - 2$
Correct Ans: A	

Itemcode : HT1024	
Q24: The value of $\lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=0}^{2n} k^2$ is	
A	1/3
B	2/3
C	4/3
D	8/3
Correct Ans: D	

Itemcode : HT1025	
Q25: Let a, b be distinct real numbers, then the number of distinct real roots of the equation $(x - a)^5 + (x - b)^5 = 0$ is	
A	1
B	2
C	3
D	Depends on the value of a, b
Correct Ans: A	

Itemcode : HT1026	
Q26: Consider the function $f(x) = \cos(x - 3) + x^2 - 1 $, then what are the points at which the function f is not differentiable?	
A	3
B	1, -1
C	Differentiable every where
D	No where differentiable
Correct Ans: B	

Itemcode : HT1027	
Q27: The value of improper integral $\int_0^{\infty} (3 + x^2)e^{-x} dx$ is	
A	5
B	

	3
C	-2
D	0
Correct Ans: A	

Itemcode : HT1028	
Q28: Consider the map $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $f(x, y) = (x^3 + 4y, x^2 + 3x + y^4)$, then	
A	The function f is not continuous at $(0,0)$.
B	The function f is continuous at $(0,0)$ but not differentiable at $(0,0)$.
C	The function f is differentiable at $(0,0)$ but the derivative $Df(0,0)$ is not invertible.
D	The function f is differentiable at $(0,0)$ and the derivative $Df(0,0)$ is invertible.
Correct Ans: D	

Itemcode : HT1029	
Q29: Find the volume of the parallelopiped whose coterminus edges are represented by the vectors $\vec{a} = 3\hat{i} - 7\hat{j} - 5\hat{k}, \vec{b} = \hat{i} + 4\hat{j} - 7\hat{k}, \vec{c} = 3\hat{i} + \hat{j} + \hat{k}$ is	
A	242
B	0
C	55
D	33
Correct Ans: A	

Itemcode : HT1030	
Q30: Which one of the following statements is true?	
A	$A = \emptyset$ implies $P(A) = \emptyset$.
B	For every set A , there exists $f: A \rightarrow P(A)$, such that f is onto.
C	For every set A , there exists $f: A \rightarrow P(A)$, such that f is one – one.
D	$\mathbb{N}$ and $\mathbb{Z}$ are perfect sets.
Correct Ans: C	

Itemcode : HT1031	
Q31: Number of fixed points of the function $f(x) = \log_e x + x - 5$ are	
A	4
B	6
C	3
D	2
Correct Ans: A	

Itemcode : HT1032	
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Q32: Which of the following limit exists?	
A	$\lim_{n \rightarrow 0} \sin\left(\frac{1}{n}\right)$
B	$\lim_{n \rightarrow 0} e^{\left(\frac{1}{n}\right)}$
C	$\lim_{x \rightarrow 0} f(x)$ where $f: [0,1] \rightarrow \mathbb{R}$ is given by $f(x) = \begin{cases} p \sin\left(\frac{1}{q}\right), & x = \frac{p}{q} \text{ in } [0,1] \\ x, & \text{otherwise} \end{cases}$
D	$\lim_{x \rightarrow 0} \frac{\cos x}{x}$
Correct Ans: C	

Itemcode : HT1033	
Q33: Choose the incorrect statement.	
A	There are uncountably many sequences converging to the limit point of a subset $S \subseteq \mathbb{R}$.
B	For monotonic functions, the set of points where limit does not exist is uncountable.
C	$\lim_{x \rightarrow \pi} \frac{ \sin x }{\sin x}$ fails to exist.
D	$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$.
Correct Ans: B	

Itemcode : HT1034	
Q34: Choose the incorrect statement.	
A	All sequences are continuous.
B	A function is continuous on an interval I , such that $f(I)$ is a subset of the set of rational numbers, then f is constant.
C	$A = \{f: \mathbb{R} \rightarrow \mathbb{R} \text{ is continuous} \mid f \text{ vanishes only on set of rational numbers}\} \neq \emptyset$.
D	Continuity is sufficient condition for Intermediate Value Property, not necessary.
Correct Ans: C	

Itemcode : HT1035	
Q35: Which of the following statements is false?	
A	$f(x) = \sin x^2$, $x \in \mathbb{R}$ is continuous, but not uniformly continuous.
B	$f(x) = e^x$, $x \in \mathbb{R}$ is continuous, but not uniformly continuous.
C	$f(x) = x $, $x \in \mathbb{R}$ is continuous, but not uniformly continuous.
D	Lipschitz condition implies uniform continuity.
Correct Ans: C	

Itemcode : HT1036	
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Q36: Choose the correct statement.	
A	$f_n(x) = \frac{x}{1+nx}, x \in [0, \infty)$ is uniformly convergent.
B	$f_n(x) = x^n, x \in [0, 1]$ is uniformly convergent.
C	$f_n(x) = \begin{cases} n^2x, & 0 \leq x < \frac{1}{n} \\ -n^2x + 2n, & \frac{1}{n} \leq x < \frac{2}{n} \\ 0, & \frac{2}{n} \leq x \leq 1 \end{cases}$ is uniformly convergent.
D	$f_n(x) = \frac{nx}{1+n^2x}, x \in [0, \infty)$ is not uniformly convergent.
Correct Ans: A	

Itemcode : HT1037	
Q37: Which one of the following statements is false?	
A	Number of linearly independent solutions for the system $Ax = 0$ is nullity $\eta(A)$.
B	If A is $n \times m$ matrix and b is $n \times 1$ vector, consider $Ax = b$, if rank $\rho(A:b) = m$. Then $Ax = b$ has at most one solution.
C	Existence of solutions of system of linear equations $Ax = 0$ depends upon rank $\rho(A)$.
D	Let $A_{n \times m} = [c_1, c_2, \dots, c_m]$, $c_i \in F^n$. Then consider $Ax = b$, $S_1 = [c_1, c_2, \dots, c_m, b]$. If S_1 is linearly dependent, then $Ax = b$ has a solution.
Correct Ans: D	

Itemcode : HT1038	
Q38: Pick the true statement.	
A	$A_{3 \times 3}$ such that $A^3 = -I$. Then, A has 3 distinct eigenvalues
B	If V is a vector space over the field $\mathcal{F}$ with $\dim V = n$, then $T: V(\mathcal{F}) \rightarrow V(\mathcal{F})$ is diagonalizable implies that all the eigenvalues are distinct.
C	T is diagonalizable implies arithmetic and geometric multiplicities for each eigenvalue of T are equal.
D	The characteristic polynomial of T splits over $\mathcal{F}$ implies T is diagonalizable.
Correct Ans: C	

Itemcode : HT1039	
Q39: Which statement is false?	
A	If $A_{n \times n}$ is an Idempotent matrix, $\rho(A) = \text{Tr}(A)$.
B	An Involutory matrix is diagonalizable over every field.
C	$a = (a_1, a_2, \dots, a_n)$, $b = (b_1, b_2, \dots, b_n) \in \mathcal{F}^n$, $A = a^T b$, $a \neq 0, b \neq 0$. Then A is diagonalizable iff $\text{rank}(A) \neq 0$.
D	Every odd skew-symmetric matrix is singular.

Correct Ans: **B**

Itemcode : **HT1040**

Q40: Pick the false statement.

- A** $A_{n \times n}$ real matrix, then every eigen value of $A^T A$ is non-negative real.
- B** $A \neq 0$ is real symmetric matrix, $Tr(A) = 0$. Then $q(x) = x^T A x$ is indefinite.
- C** $A_{n \times n}$ real symmetric non-singular matrix, such that $\exists x \in \mathbb{R}^n$ s.t. $x^T A x < 0$. Then $B = -A$ is positive-definite.
- D** For A a real symmetric matrix, the range of $q(x) = x^T A x$ is a subset of $\mathbb{R}^- \cup \{0\}$ for q negative-definite.

Correct Ans: **C**

Itemcode : **HT1041**

Q41: Which of the following is incorrect?

- A** For $A_{n \times m}, B_{m \times n}, X = AB, Y = BA$. Then $|X| = |Y|$.
- B** $T: V \rightarrow V, V = W_1 \oplus W_2$ where both W_1 and W_2 are T invariant subspaces. Then $m_T(x) = LCM\{m_{T_1}(x), m_{T_2}(x)\}$ where T_i are induced operators on W_i by T .
- C** Let a matrix A has Jordan-canonical form, then the number of Jordan blocks corresponding to eigenvalue λ_i = Geometric multiplicity of λ_i .
- D** In the Jordan-canonical form, the sum of the order of Jordan-blocks corresponding to eigenvalue λ_i = Arithmetic multiplicity of λ_i .

Correct Ans: **A**

Itemcode : **HT1042**

Q42: $\frac{dy}{dx} = ay - by^2; a, b > 0$ $y(0) = y_0$. Then as $x \rightarrow \infty$, $y(x)$ tends to

- A** 0
- B** $\frac{a}{b}$
- C** $\frac{b}{a}$
- D** y_0

Correct Ans: **B**

Itemcode : **HT1043**

Q43: Consider $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$; $a_0 \neq 0$; a_0, a_1, a_2 are continuous functions, y_1 and y_2 are 2 linearly independent solutions of the above equation. Which of the following statement is false?

- A** The sets of zeroes of y_1 and y_2 are disjoint.
- B** The set of zeroes is dense in $\mathbb{R}$.
- C** The set of zeroes of y_1 is finite iff the set of zeroes of y_2 is finite.

D	If $\alpha_0 < \alpha_1$ for α_0, α_1 in the set of zeroes of y_1 , then $\exists \beta$, a zero of y_2 such that $\alpha_0 < \beta < \alpha_1$.
Correct Ans: B	

Itemcode : HT1044	
Q44: The singular solution of $(xp - y)^2 = p^2 - 1$ is	
A	$x^2 + y^2 = 1.$
B	$x^2 - y^2 = 1.$
C	$x^2 + 2y^2 = 1.$
D	$2x^2 + y^2 = 1.$
Correct Ans: B	

Itemcode : HT1045	
Q45: The general solution for $yp - xq = yz^2 e^{-(x^2+y^2)}$ is	
A	$u = x^2 + y^2, v = x + \frac{1}{z} e^{(x^2+y^2)}.$
B	$u = x^2 + y^2, v = x - \frac{1}{z} e^{-(x^2+y^2)}.$
C	$u = x^2 - y^2, v = x + \frac{1}{z} e^{(x^2+y^2)}.$
D	$u = x^2 - y^2, v = x - \frac{1}{z} e^{-(x^2+y^2)}.$
Correct Ans: A	

Itemcode : HT1046	
Q46: The complete integral of $(p^2 + q^2)y - qz = 0$ is	
A	$z^2 = a^2 y^2 + (ax + b)^2.$
B	$z^2 = ay^2 + (ax + b)^2.$
C	$z^2 = a^2 y + (ax - b)^2.$
D	$z = a^2 y^2 + (ax + b)^2.$
Correct Ans: A	

Itemcode : HT1047	
Q47: $(x-1)^2 u_{xx} - (y-2)^2 u_{yy} + 2xu_x + 2yu_y + 2xyu = 0$ is parabolic in $S \subseteq \mathbb{R}^2$ but not in $\mathbb{R}^2 \setminus S$. Then S is	
A	$\{(x, y) \in \mathbb{R}^2: x = 1 \text{ or } y = 2\}.$
B	$\{(x, y) \in \mathbb{R}^2: x = 1 \text{ and } y = 2\}.$
C	

	$\{(x, y) \in \mathbb{R}^2 : x = 1\}.$
D	$\{(x, y) \in \mathbb{R}^2 : y = 2\}.$
Correct Ans: A	

Itemcode : HT1048	
Q48: Choose incorrect statement.	
A	A group G is abelian if and only if every element in G is self-inverse.
B	G is an abelian group if and only if $(ab)^2 = a^2b^2 \forall a, b \in G.$
C	The identity element e is the only element with unit order.
D	Smallest non-abelian group of even order is a group of order 6.
Correct Ans: A	

Itemcode : HT1049	
Q49: Choose incorrect statement.	
A	Number of generators of the group $\mathbb{Z}_{200}$ is 80.
B	$U(9)$ is a cyclic group.
C	Every subgroup other than Q_8 of Q_8 is abelian.
D	Every group can be expressed as the union of its two proper subgroups.
Correct Ans: D	

Itemcode : HT1050	
Q50: Choose the correct statement.	
A	A group has an element of order k if and only if there is a subgroup of order k .
B	A group of order 20 can have two distinct subgroups of order 5.
C	A cycle in S_n can be expressed as a product of transpositions in infinitely many ways.
D	Number of 4-cycles in 4-symbols = 4.
Correct Ans: C	

Itemcode : HT1051	
Q51: Which one of the following statements is correct?	
A	$\mathbb{Z}_2 \times \mathbb{Z}_4$ is cyclic.
B	Image of an abelian group under any group homomorphism is abelian.
C	Number of group homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{18}$ is 4
D	If G is a non-abelian group of order p^3 then the number of conjugate classes of G is given by $p^2 - p + 1$.
Correct Ans: B	

Itemcode : HT1052	
Q52: Which one of the following is incorrect?	
A	$(\mathbb{C}^*, \cdot)$ has an element of order n for every n .
B	

	$(\mathbb{C}^*, \cdot)$ is cyclic.
C	If $x \in \mathbb{C}^*, o(x) < \infty \Rightarrow x = e^{i\theta}$.
D	$(\mathbb{R}^*, \cdot)$ has exactly 2 elements of finite order.
Correct Ans: B	

Itemcode : HT1053	
Q53: Choose the incorrect statement.	
A	Composition of group automorphisms is also an automorphism.
B	$Aut(K_4) \cong S_3$.
C	$o(SL(2, \mathbb{Z}_5)) = 100$.
D	$Aut(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2) \cong GL(3, \mathbb{Z}_2)$.
Correct Ans: C	

Itemcode : HT1054	
Q54: Choose the incorrect statement.	
A	$\mathbb{Z}_7$ is a simple group.
B	Every group G with $ G = p^3$ is simple.
C	Quotient group of an infinite group can be finite.
D	If G is non-abelian group of order p^3 , then $\frac{G}{Z(G)} \cong \mathbb{Z}_p \times \mathbb{Z}_p$.
Correct Ans: B	

Itemcode : HT1055	
Q55: Pick out the false statement.	
A	$\frac{\mathbb{Q}}{\mathbb{Z}}$ is an infinite group each of whose element is of finite order.
B	$\frac{\mathbb{Q}}{\mathbb{Z}}$ has exactly one cyclic subgroup of order n for every n .
C	D_5 has 5 elements of order 2.
D	Number of conjugate classes for $D_8 = 8$.
Correct Ans: D	

Itemcode : HT1056	
Q56: Which one of the following statements is incorrect?	
A	For $ G = 200$, there exists unique subgroup of order 25.
B	A group with $ G = 15$ is cyclic.
C	A group with $ G = 1001$ is simple.
D	A group G with order 108 is not simple.
Correct Ans: C	

Itemcode : HT1057

Q57: Which of the following statements is incorrect?

- | | |
|---|---|
| A | Compact subset of every topological space is closed. |
| B | Sequentially compact metric space is totally bounded. |
| C | Discrete space with more than one point is disconnected. |
| D | If X is a metric space, S is a connected subset, $S \subseteq A \subseteq \bar{S}$ then A is connected. |

Correct Ans: A

Itemcode : HT1058

Q58: $[a, b] \subseteq \mathbb{R}$, f be a continuous function, then $f([a, b])$ is

- | | |
|---|-------------------------|
| A | Open and bounded. |
| B | Closed but not bounded. |
| C | Disconnected |
| D | Closed and bounded. |

Correct Ans: D

Itemcode : HT1059

Q59: Which one of the following subsets of $\mathbb{R}^3$ is compact under usual topology?

- | | |
|---|---|
| A | $\{(x_1, x_2, x_3) : x_i < 1, 1 \leq i \leq 3\}$ |
| B | $\{(x_1, x_2, x_3) : x_1 + x_2 + x_3 = 0\}$ |
| C | $\{(x_1, x_2, x_3) : x_i \geq 0, 1 \leq i \leq 3\}$ |
| D | $\{(x_1, x_2, x_3) : x_i \leq 2, 1 \leq i \leq 3\}$ |

Correct Ans: D

Itemcode : HT1060

Q60: Let A be an $n \times n$ matrix over $\mathbb{R}$. Then which one of the following is false?

- | | |
|---|--|
| A | Every solution of the system $A^T A X = 0$ is also a solution of $A X = 0$. |
| B | The systems $A X = 0$ and $A^T A X = 0$ have the same set of solutions. |
| C | $\text{Rank}(A^T A) = \text{Rank}(A)$. |
| D | There exists $x_0 \in \mathbb{R}^n$ such that $A x_0 = 0$ and $A^T A x_0 \neq 0$. |

Correct Ans: D

Itemcode : HT1061

Q61: Let V be the set of all continuous functions on $[a, b]$. Then

- | | |
|---|---|
| A | V is not a vector space. |
| B | $\{\sin x, \sin^2 x, \sin^3 x\}$ is linearly independent in V . |
| C | $\{1, \cos x, \cos^2 x\}$ is linearly dependent in V . |

D	V is a finite dimensional vector space.
Correct Ans: B	

Itemcode : HT1062	
Q62: Let $A \in M_n(\mathbb{R})$ such that $A = A^T$. Then which one is incorrect?	
A	Every eigenvalue of A is real.
B	If $x_1, x_2 \in \mathbb{R}^n$ are eigenvectors of A corresponding to distinct eigenvalues, then $x_1^T x_2 = 0$.
C	Every eigenvalue of A is positive.
D	There is an orthonormal set $\{x_1, x_2, \dots, x_n\}$ in $\mathbb{R}^n$ such that $A x_i = \lambda_i x_i, i = 1, 2, \dots, n$ for some $\lambda_i \in \mathbb{R}$.
Correct Ans: C	

Itemcode : HT1063	
Q63: Let $A \in M_{m \times n}(\mathbb{C}), B \in M_{n \times m}(\mathbb{C})$. Then	
A	BA and AB have same set of eigenvalues only if $\text{Rank}(AB) = \text{Rank}(BA)$.
B	AB is diagonalizable iff BA is diagonalizable.
C	If $(x - \lambda)^r$ divides the characteristic polynomial of AB for some $\lambda \in \mathbb{C}, r \in \mathbb{N}$, then $(x - \lambda)^r$ divides the characteristic polynomial of BA .
D	If $(x - \lambda)^r$ divides the characteristic polynomial of BA for some $\lambda \in \mathbb{C}, \lambda \neq 0, r \in \mathbb{N}$, then $(x - \lambda)^r$ divides the characteristic polynomial of AB .
Correct Ans: D	

Itemcode : HT1064	
Q64: If A is a 3×3 matrix having eigenvalues 0,2,3 with eigenvectors u, v, w respectively, then	
A	The linear system $Ax = u$ is consistent.
B	The linear system $Ax = v + w$ is consistent.
C	The linear system $Ax = u$ has infinitely many solutions.
D	The linear system $Ax = v + w$ has a unique solution.
Correct Ans: B	

Itemcode : HT1065	
Q65:	
If $A = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$, then	
A	$ A = -1$.
B	A is diagonalizable over $\mathbb{R}$.
C	Minimal polynomial of A is $x^4 + x^3 + x^2 + x + 1$.
D	A has a rational eigenvalue.

Correct Ans: **C**

Itemcode : **HT1066**

Q66: If $f(z) = (e^z - i)^{-1}$, for all $z \in \mathbb{C}$ such that $e^z \neq i$, then which one of the following is correct?

- A** f is entire.
- B** All the poles of f lie on the imaginary axis.
- C** f has a double pole.
- D** All the poles of f lie on the real axis.

Correct Ans: **B**

Itemcode : **HT1067**

Q67: If $I_\gamma = \int_\gamma \frac{dz}{z}$, then

- A** $I_\gamma = \log \sqrt{2} + i\frac{\pi}{4}$ if γ is the straight line from 1 to $1 + i$.
- B** $I_\gamma = 0$ if γ is the unit circle .
- C** $I_\gamma = 2\pi i$ if γ is the circle with center $z = 3$ and radius 1.
- D** $I_\gamma = 0$ for any closed curve γ in the complex plane.

Correct Ans: **A**

Itemcode : **HT1068**

Q68: The conformal map $w = z^2$ takes the half disk $\{z: |z| < 1, \text{Im } z > 0\}$ onto

- A** $\{w: |w| < 1\}$
- B** $\{w: \text{Re } w > 0\}$
- C** $\{w: |w| < 1, 0 < \arg w \leq 2\pi\}$
- D** $\{w: |w| < 1, 0 < \arg w < 2\pi\}$

Correct Ans: **D**

Itemcode : **HT1069**

Q69: $\int_\gamma (z^2 - 3|z| + \text{Im } z) dz$, where γ is the quarter circle centered at the origin and extending from 2 to $2i$ is

- A** 0
- B** $\frac{28}{3} - \pi + \frac{38}{3}i$
- C** $2\pi i$
- D** $\frac{28}{3} - \pi - \frac{38}{3}i$

Correct Ans: **D**

Itemcode : **HT1070**

Q70: For the function $f(z) = \frac{1+z}{1-z^4}$, which one of the following is true?

- A** All the singularities of f are poles.
- B** f has a removable singularity.
- C** f has an essential singularity.
- D** All the poles of f are of order 2.

Correct Ans: **B**

Itemcode : **HT1071**

Q71: The map $f(z) = \frac{e^z - 1}{e^z + 1}$, maps the horizontal strip $S = \left\{ z: \frac{-\pi}{2} < \text{Im } z < \frac{\pi}{2} \right\}$ conformally onto

- A** the right half plane.
- B** the horizontal strip S .
- C** the unit disk $\mathbb{D}$.
- D** the first quadrant.

Correct Ans: **C**

Itemcode : **HT1072**

Q72: Let $S \subseteq \mathbb{R}$ such that $\text{Int}(S) \neq \emptyset$. Then which of the following is false?

- A** S has a convergent sequence of real numbers.
- B** S is uncountable.
- C** S contains a closed interval as a proper subset.
- D** S is a countable union of disjoint intervals.

Correct Ans: **D**

Itemcode : **HT1073**

Q73: If f and g are functions on $\mathbb{R}$, then which of the following is true?

- A** If $f \circ g = g \circ f$, then $f = g$.
- B** If $f \circ g = g \circ f$, then either f or g is an identity function.
- C** If A and B are subsets of $\mathbb{R}$, then $f(A \cup B) = f(A) \cup f(B)$.
- D** If A and B are subsets of $\mathbb{R}$, then $f(A \cap B) = f(A) \cap f(B)$.

Correct Ans: **C**

Itemcode : **HT1074**

Q74: Consider the sequence $a_n = (-1)^n \frac{n+1}{n}$, $n \geq 1$. Let $l = \liminf a_n$ and $s = \limsup a_n$. Choose the correct statement from below.

A	$l = s = 0.$
B	$-1 \leq l < s \leq 1.$
C	$\sum_{n=1}^{\infty} a_n$ is convergent.
D	$\langle a_n \rangle$ is convergent.
Correct Ans: B	

Itemcode : HT1075	
Q75: Let $\mathbb{R}^2 = \{(x_1, x_2) : x_1, x_2 \in \mathbb{R}\}$ and $\langle \cdot, \cdot \rangle : \mathbb{R}^2 \rightarrow \mathbb{R}$ be an inner product on $\mathbb{R}^2$ defined by $\langle x, y \rangle = x_1 y_1 + x_2 y_2$ for $x = (x_1, x_2), y = (y_1, y_2)$. Then $\sup \{ \langle x, y \rangle + \langle y, z \rangle + \langle z, x \rangle : \langle x, x \rangle = \langle y, y \rangle = \langle z, z \rangle = 1 \}$ is	
A	3
B	4
C	2
D	does not exist
Correct Ans: A	

Itemcode : HT1076	
Q76: Let Y be the set of all real-valued continuous functions defined on $[0, 1]$ with supnorm. Let X be the subspace of all continuously differentiable functions in Y . Define $T: X \rightarrow Y$ by $T(x(t)) = x'(t)$. Then	
A	T is not a linear transformation.
B	T is bounded.
C	Graph(T) is closed in $X \times Y$.
D	T is continuous.
Correct Ans: C	

Itemcode : HT1077	
Q77: Let $B(H)$ be the set of all bounded linear transformations defined on a Hilbert space H , and S be the set of all self-adjoint operators in $B(H)$. Then	
A	S is a closed subset of $B(H)$.
B	S is not a subspace of $B(H)$ over $\mathbb{R}$.
C	S is an empty set.
D	S is neither open nor closed.
Correct Ans: C	

Itemcode : HT1078	
Q78: Let $\mathbb{Q}$ be the set of all rational numbers and $I = [0, 1]$.	
A	(a) Lebesgue outer measure of $I \cap \mathbb{Q}$ is $\frac{1}{2}$.
B	(b) Lebesgue outer measure of $I \cap \mathbb{Q}^c$ is $\frac{1}{2}$.
C	(c) Let G be the set of all numbers in $[0, 1]$ which possesses decimal expansions not containing the digit 5 has measure $\frac{4}{10}$.

D	(d) The set G defined in (c) has measure 0.
Correct Ans: D	

Itemcode : HT1079	
Q79: Let τ_1 be the standard topology on $\mathbb{R}$ and τ_2 be the cofinite topology on $\mathbb{R}$. Then	
A	$(\mathbb{R}, \tau_1)$ is finer than $(\mathbb{R}, \tau_2)$.
B	$(\mathbb{R}, \tau_2)$ is finer than $(\mathbb{R}, \tau_1)$.
C	Both $(\mathbb{R}, \tau_1)$ is finer than $(\mathbb{R}, \tau_2)$. and $(\mathbb{R}, \tau_2)$ is finer than $(\mathbb{R}, \tau_1)$.
D	$(\mathbb{R}, \tau_1)$ and $(\mathbb{R}, \tau_2)$ are not comparable.
Correct Ans: D	

Itemcode : HT1080	
Q80: Let $(\mathbb{R}^2, \tau)$ be a topological space with standard topology τ and $A = \{(a, b) \in \mathbb{R}^2 : \sin a = 0 \text{ and } b \in \mathbb{Q}\}$. Then	
A	$\mathbb{R}^2 - A$ is path-wise connected.
B	$\mathbb{R}^2 - A$ is disconnected.
C	If $B \subseteq \mathbb{R}^2$ is connected, then $\mathbb{R}^2 - B$ is path-wise connected.
D	A is a closed subset in $\mathbb{R}^2$.
Correct Ans: A	

Itemcode : HT1081	
Q81: To which district of H.P. did Jaiwant Ram, the first Speaker of H.P. Legislative Assembly, belong?	
A	Bilaspur
B	Mandi
C	Chamba
D	Sirmour
Correct Ans: C	

Itemcode : HT1082	
Q82: Which Assembly constituencies of Bilaspur district of H.P. were initially represented by a husband-wife team during 1951-54 elections?	
A	Ghumarwin and Gehrwin
B	Gehrwin and Kot Kehlur
C	Kot Kehlur and Bilaspur
D	Bilaspur and Ghumarwin
Correct Ans: A	

Itemcode : HT1083	
Q83: Which one of the following was a double-member constituency during the 1951-54 Vidhan Sabha elections in H.P.?	

A	Theog
B	Rajgarh
C	Jubbal
D	Kasumpti
Correct Ans: A	

Itemcode : HT1084	
Q84: The inquiry Commission set-up in 2012 to look into all benami land deals in H.P. was headed by Justice _____.	
A	V.K. Sharma
B	D.P. Sood
C	Sandeep Sharma
D	S.S. Negi
Correct Ans: B	

Itemcode : HT1085	
Q85: Approximately, what percentage of total geographical area of the country falls in H.P.?	
A	1.2
B	1.7
C	2.3
D	2.5
Correct Ans: B	

Itemcode : HT1086	
Q86: What was the per capita income in H.P. (in rupees at current prices) during 1950-51?	
A	160
B	180
C	240
D	290
Correct Ans: C	

Itemcode : HT1087	
Q87: When was Rajiv Gandhi <u>Ann Yojna</u> introduced in H.P.?	
A	2012
B	2013
C	2014
D	2015
Correct Ans: B	

Itemcode : HT1088	
Q88: Which district of H.P. is the main beneficiary of Shah canal?	
A	Bilaspur

B	Hamirpur
C	Kangra
D	Una
Correct Ans: C	

Itemcode : HT1089	
Q89: How many districts of H.P. are excluded from Mid-Himalayan Watershed Project?	
A	One
B	Two
C	Three
D	Four
Correct Ans: B	

Itemcode : HT1090	
Q90: To which district of H.P. does Charanjit Singh, who was captain of Indian Hockey Team during the 1964 Tokyo Olympics, belong?	
A	Bilaspur
B	Una
C	Kangra
D	Chamba
Correct Ans: B	

Itemcode : HT1091	
Q91: Sania Mirza is associated with _____.	
A	Badminton
B	Hockey
C	Lawn Tennis
D	Soft Ball
Correct Ans: C	

Itemcode : HT1092	
Q92: Where is the Headquarters of Southern Command of Indian Army?	
A	Secundrabad
B	Pune
C	Kochi
D	Vizag
Correct Ans: B	

Itemcode : HT1093	
Q93: What is the upper age limit for eligibility to Accident Insurance under Pradhan Mantri Suraksha Bima Yojna?	
A	60 years
B	65 years

C	70 years
D	75 years
Correct Ans: C	

Itemcode : HT1094	
Q94: Which one of the following is <u>NOT</u> included in Tertiary sector of Indian economy?	
A	Animal Husbandry
B	Banking
C	Tourism
D	Insurance
Correct Ans: A	

Itemcode : HT1095	
Q95: Who is the author of <u>Geet Gobind</u> ?	
A	Jai Dev
B	Patanjali
C	Asvaghosh
D	Kali Das
Correct Ans: A	

Itemcode : HT1096	
Q96: What is the currency of Iraq?	
A	Yen
B	Dinar
C	Rial
D	Dirham
Correct Ans: B	

Itemcode : HT1097	
Q97: Who propounded the Human Development Index (HDI)?	
A	Ranjit Guha
B	Amartya Sen
C	Mahbub-ul-Haq
D	Urjit Patel
Correct Ans: C	

Itemcode : HT1098	
Q98: Among the following, who was given Nobel Prize for Peace in 2014?	
A	V.S. Naipaul
B	V. Ramakrishnan
C	Kailash Satyarthi
D	None of these

Correct Ans: C

Itemcode : HT1099	
Q99: Suez canal connects Mediterranean Sea and _____.	
A	Baltic Sea
B	Red Sea
C	Caspian Sea
D	Dead Sea
Correct Ans: B	

Itemcode : HT1100	
Q100: Given below are the names of some countries and their capitals. Find the mis-match.	
A	Angola - Luanda
B	Egypt - Cairo
C	Uganda - Ankara
D	Sudan - Khartoum
Correct Ans: C	